Advanced soil models.
Calibration of Modified ECP-Hujeux model

21.08.2007

Introduction

- The multi-mechanism elasto-plastic model (Aubry 1982, Hujeux 1985) is designed to properly reproduce soil behavior subject to monotonic and cyclic loads
- The original version of the model consists of three plane-strain deviatoric plastic mechanisms in three orthogonal planes and a single volumetric mechanism
- Hardening laws for deviatoric mechanisms: mixed isotropic/kinematic type
- Major extension of the basic Aubry's model, made by Hujeux, is related to the introduction of so-called domains of soil behavior for the three deviatoric mechanisms
  - Linear domain: $0 \leq \gamma_c \leq 10^{-5}$
  - Stabilized hysteretic domain: $10^{-5} \leq \gamma_c \leq 10^{-4}$
  - Nonstabilized hysteretic domain: $10^{-4} \leq \gamma_c \leq 5 \times 10^{-3}$
  - Domain of incremental laws: $\gamma_c \geq 5 \times 10^{-3}$

Introduction

- In the current version of Z_Soil the modified version of the model is implemented
- 3 deviatoric yield surfaces are replaced by one which includes an effect of strength anisotrophy in the deviatoric plane
- Cyclic mechanisms are not included

Notation: Stress and strain quantities

- $p = -\frac{1}{3}\sigma_{kk}$
- $s_{ij} = \sigma_{ij} + p\delta_{ij}$
- $q = \left(\frac{3}{2}s_{ij}s_{ij}\right)^{\frac{1}{2}}$
- $\varepsilon_v = \varepsilon_{kk}$
- $e_{ij} = \varepsilon_{ij} - \frac{1}{3}\varepsilon_v\delta_{ij}$
- $\dot{\varepsilon}_D = \left(\frac{2}{3}\dot{e}_{ij}\,\dot{e}_{ij}\right)^{\frac{1}{2}}$

NB. Indices i, j, k vary from 1 to 3

Nonlinear elastic behavior

The recoverable part of the deformation is driven by the initial shear G and bulk K moduli expressed by the following power laws:

$$G = G_a \left(\frac{p}{p_{\text{ref}}}\right)^n \quad K = K_a \left(\frac{p}{p_{\text{ref}}}\right)^n$$

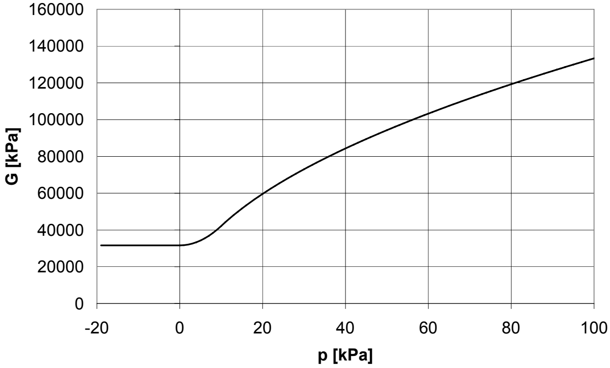
$$\frac{G}{K} = \frac{G_a}{K_a} = \text{const} = \frac{3}{2}\,\frac{1-2\nu}{1+\nu}$$

From the algorithmic point of view it may happen that p can be negative, hence the following regularization is used:

$$\begin{array}{lll} G = G_a \left(\dfrac{p}{p_{\text{ref}}}\right)^n & \text{if} & p \geq p_L \\ G = \overline{G} + A\,p^2 & \text{if} & 0 \leq p < p_L \\ G = \overline{G} & \text{if} & p < 0 \end{array}$$

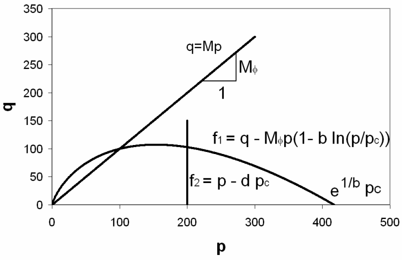
Nonlinear elastic behavior

Example of $G(p)$ function
$(G_a = 10000$ kPa, $p_{\mathrm{ref}} = 1$ kPa, $n = 0.5$, $p_L = 10$ kPa

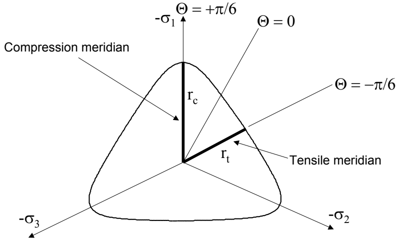


NB. This regularization preserves continuity of the $G(p)$ formula up to first derivative which is crucial for the implicit implementation

Yield surfaces



(a) Section by p-q plane



(b) Section by deviatoric plane

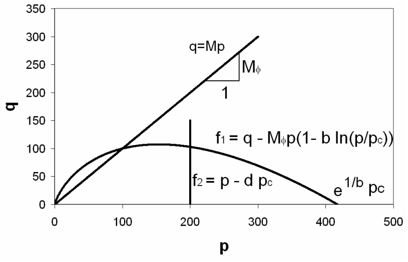
$$f_1^m = q - M_\phi\, p \left(1 - b\, ln\left(\frac{p}{p_c}\right)\right)\, r_1^m \qquad f_2^m = p - d\, p_c\, r_2^m$$

$$M_\phi = M_\phi^c\;\, g(\theta)$$

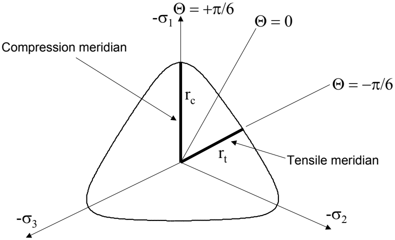
$$g\,(\theta) = \left(\frac{1 - \alpha\, \sin(3\theta)}{1 - \alpha}\right)^m \qquad sin(3\theta) = -\frac{3\sqrt{3}}{2}\frac{J_3}{J_2^{\frac{3}{2}}} \qquad m = -0.229$$

$$\alpha = \frac{k^{\frac{1}{n}} - 1}{k^{\frac{1}{n}} + 1} \leq 0.7925$$

Yield surfaces (continuation)



(a) Section by p-q plane



(b) Section by deviatoric plane

$$f_1^m = q - M_\phi \; p \left(1 - b \; ln\left(\frac{p}{p_c}\right)\right) \; r_1^m \qquad f_2^m = p - d \; p_c \; r_2^m$$

- r_1^m, r_2^m are hardening parameters for deviatoric and volumetric mechanism
- p_c is the critical pressure
- M_ϕ^c is the slope of critical state line measured along compression meridian $M_\phi^c = \dfrac{6 \; sin(\phi)}{3 - sin\phi}$

Plastic flow rule for deviatoric mechanism

- Plastic flow rule is assumed to be associated in the deviatoric plane while the corresponding volumetric part is replaced by an enhanced counterpart; the flow vector $\mathbf{b}_1$ is defined as follows

$$\mathbf{b}_1^m = \text{dev}\left(\frac{\partial f_1^m}{\partial \boldsymbol{\sigma}}\right) - \frac{1}{3}\mathbf{1}\ \psi_v^m \qquad \text{and} \qquad \psi_v^m = M_\psi - \frac{q}{p}$$

- By analogy to the expression for M_ϕ $\quad M_\psi = M_\psi^c\ g(\theta)$ and

$$M_\psi^c = \frac{6\ sin(\psi)}{3 - sin(\psi)} \qquad \text{where } \psi \text{ is a dilatancy angle}$$

Remarks

- Term ψ_v^m holds true for stress states remaining on the deviatoric yield surface f_1^m
- By default $\psi = \phi$

Plastic flow rule for volumetric mechanism

- Plastic flow rule is assumed to be associated with the yield surface f_2^m
- Plastic flow vector $\mathbf{b}_2^m$ is defined as follows

$$\mathbf{b}_2^m = \frac{\partial f_2^m}{\partial \boldsymbol{\sigma}} = -\frac{1}{3}\mathbf{1}$$

Evolution law for hardening parameter r_1^m

- In the original version of the model the evolution law for r_1^m parameter is defined as follows:

$$\dot{r}_1^m = \dot{\gamma}^p \frac{(1 - r_1^m)^2}{a_m} = \dot{\lambda}_1^m \frac{(1 - r_1^m)^2}{a_m}$$

  For constant a_m material parameter integrating the above expression yields a hyperbolic dependence of r_1^m with respect to γ^p

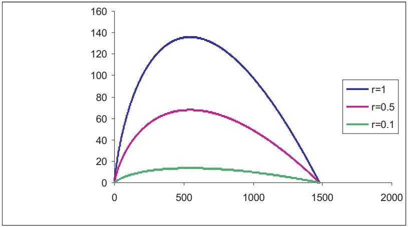
$$r_k^m = \frac{\gamma_k^p}{a_m + \gamma_k^p}$$

- Better matching of $\frac{G}{G_o}(\gamma)$ curves (after Mellal) to the real measurements are obtained for modified law

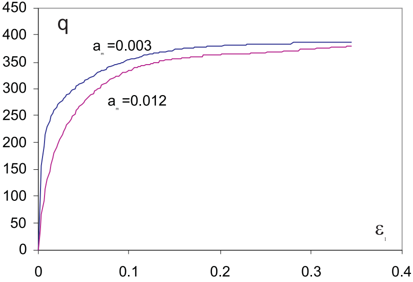
$$\dot{r}_1^m = \dot{\gamma}^p \frac{\left(1 - (r_1^m)^2\right)^2}{2\, r_1^m\, a_m} = \dot{\lambda}_1^m \frac{\left(1 - (r_1^m)^2\right)^2}{2\, r_1^m\, a_m}$$

Evolution law for hardening parameter r_1^m

r_1^m parameter reflects level of mobilization of the mechanism



Evolution of yield surface with r_1^m



Influence of a_m parameter

Evolution law for r_2^m parameter

- The evolution law for the degree of mobilization of volumetric mechanism is defined as follows:

$$\dot{r}_2^m = \dot{\lambda}_2 \frac{\left(1 - r_2^m\right)^2}{c_m}$$

- This law yields a hyperbolic dependence of r_2^m with respect to $\varepsilon_{v_2}^p$ (volumetric plastic strain generated exclusively by volumetric mechanism)
- c_m is a material parameter

NB. To simplify the model we assume $r_2^m = 1$ (mechanism is fully mobilized)

Evolution law for critical pressure p_c

- The critical pressure p_c is a hardening parameter coupling both deviatoric and volumetric mechanism
- The evolution law for p_c parameter is defined as follows:
  $\dot{p}_c = -\beta \; p_c \; \dot{\varepsilon}_v^p$
  where ε_v^p is the volumetric plastic strain rate accumulated from all active mechanisms

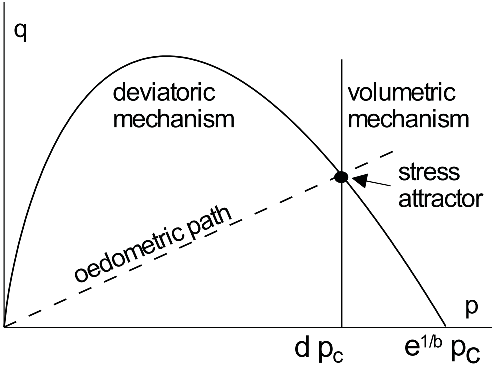
Material parameters

All material parameters are classified in the following five groups:

- Elastic parameters: G_a, ν, n
- Critical state parameters: β, ϕ, ψ
- Parameters for deviatoric hardening: a_m, b
- Parameters for volumetric hardening: c_m, d
- Limits of elastic behavior: r_1^{el}, r_2^{el}

Oedometer test

- In the oedometric test K_o ($K_o = \dfrac{\sigma_h}{\sigma_v}$) depends exclusively on ϕ, d and b parameters, and it tends to a constant value with an increasing load
- Stress path is attracted by the intersection of the deviatoric and volumetric yield surfaces



Oedometer test

Assumptions:

- With an increasing load deviatoric and volumetric mechanisms become fully mobilized ($r_1^m \rightarrow 1$ and $r_2^m \rightarrow 1$) hence in the further derivations we will explicitly set them equal to 1
- As the stress state is attracted by the intersection point, the two yield conditions must be satisfied simultaneously

$$q - p\ M_\phi\ \left(1 - b\ ln\frac{p}{p_c}\right) = 0$$

$$p - d\ p_c = 0$$

Form the second equation one can evaluate $\dfrac{p}{p_c} = d$ and then substitute it into the first equation which reads

$$q - p\ M_\phi\ \left(1 - b\ \ln\left(d\right)\right) = 0$$

Oedometer test

Derivation

- In the oedometric test $\sigma_x = \sigma_z$
  $q = (\sigma_x - \sigma_y)$
  $p = -\frac{1}{3}(\sigma_x + \sigma_y + \sigma_z) = -\frac{1}{3}(2\,\sigma_x + \sigma_y)$
- Expressing σ_x and σ_y through p and q one may find that K_o^{NC} (for state of normal consolidation), is expressed by the formula

$$K_o^{NC} = \frac{\sigma_x}{\sigma_y} = \frac{1 - \frac{1}{3}\frac{q}{p}}{1 + \frac{2}{3}\frac{q}{p}} = \frac{1 - \frac{1}{3}\xi}{1 + \frac{2}{3}\xi}$$

where

$$\xi = (1 - b\ \ln\ d)\ M_\phi^c = (1 - b\ \ln\ d)\ \frac{6\ sin(\phi)}{3 - sin(\phi)}$$

Oedometer test

Derivation

- It is possible to get an exact match between K_o^{NC} and Jaky's K_o

$$K_o^{NC} = \frac{1 - \frac{1}{3}\xi}{1 + \frac{2}{3}\xi} = 1 - \sin(\phi)$$

This way parameters b and d become dependent as follows

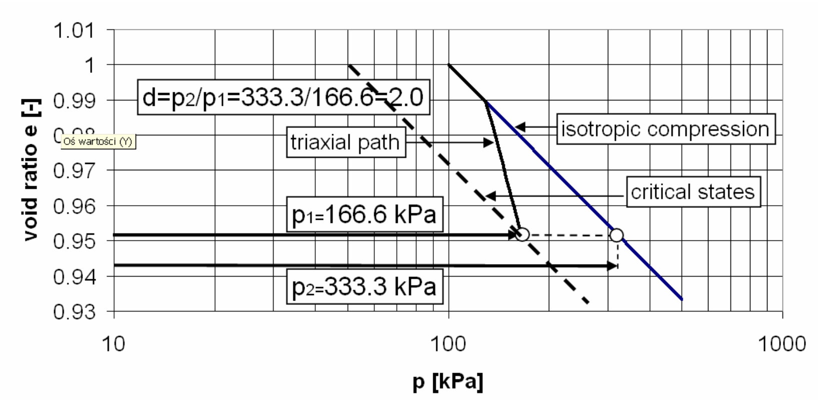
$$b = \frac{3}{2\ \ln(d)} \frac{1 - \sin(\phi)}{(3 - 2\ \sin(\phi))} \qquad d = e^{\left(\frac{3}{2b}\ \frac{1 - \sin(\phi)}{3 - 2\ \sin(\phi)}\right)}$$

- d is usually equal $d = 2$

Remark

K_o^{NC} value generated by the model is crucial for soil-structure interaction problems

Estimation of d parameter

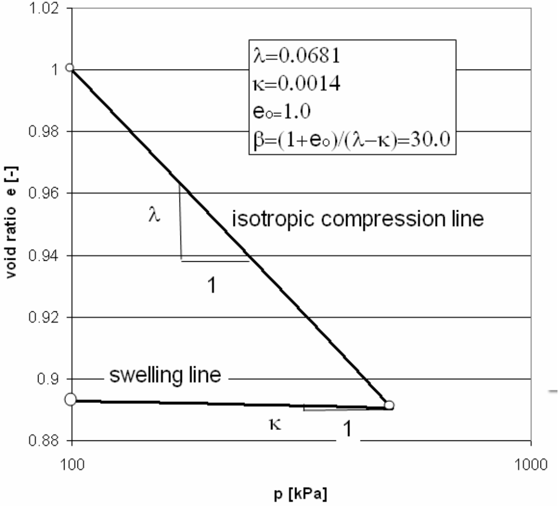


- Remark

Assuming low values of b parameter and assuming $d = 2$ (Hujeux 1985) the K_o^{NC} will be underestimated, hence b should be derived from the oedometric estimation for the assumed d value.

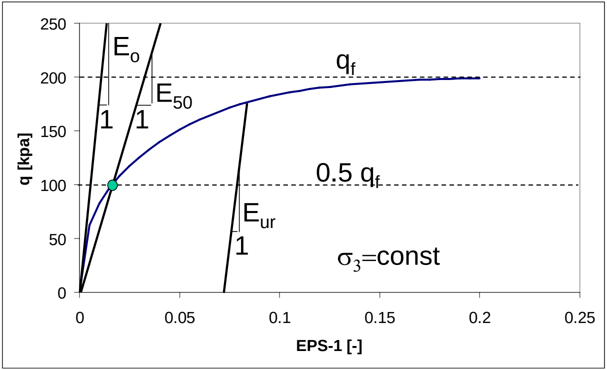
Estimation of β parameter

Isotropic compression/oedometric test



Notion of the initial/secant and unloading/reloading stiffness moduli

$$G_{ur} = \frac{E_{ur}}{2\ (1 + \nu_{ur})}$$



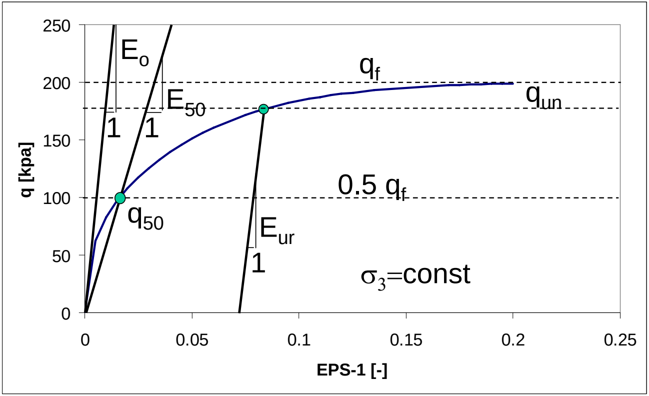
To simplify the model we assume the ν parameter to be equal to $\nu = \nu_{ur}$ and ν_{ur} usually varies in the range $\nu_{ur} = 0.2..0.3$.

G_a & n parameters in the range of small strains

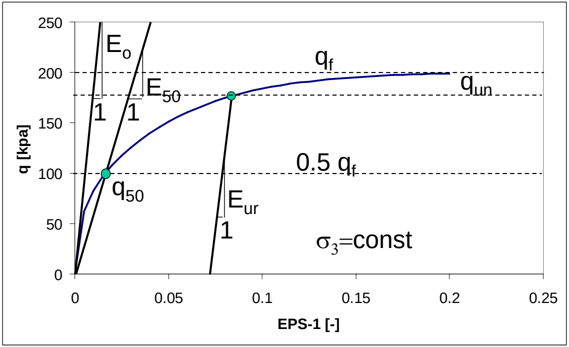
Class of soil	Void ratio e_o	Index of plasticity	G_a	n	Formula by..
Reconstituted clean sands	0.64..0.81	—	$(14300..16600)\ \frac{(2.17-e)^2}{1+e}$	0.4	Iwasaki and Tatsuoka
Undisturbed sands without fines content	0.64..0.81	—	$(7900..14300)\ \frac{(2.17-e)^2}{1+e}$	0.4	Iwasaki and Tatsuoka
Undisturbed sands with fines content above 50%	0.64..0.81	—	$(2360..3090)\ \frac{(2.17-e)^2}{1+e}$	0.6	Higuchi
Clays	0.6..1.5	low plasticity index	$3270\ \frac{(2.97-e_o)^2}{1+e_o}$	0.5	Hardin and Black
Clays	1.5..2.5	high plasticity index	$445\ \frac{(4.40-e_o)^2}{1+e_o}$	0.5	Marcuson and Wahls
Soft alluvial clays	1.5..4.0	0.4..1.00	$90\ \frac{(7.32-e_o)^2}{1+e_o}$	0.6	Kokusho
Crushed rock	0.39..0.58	—	$13000\ \frac{(2.17-e_o)^2}{1+e_o}$	0.55	Kokusho and Esashi
Round gravel	0.32..0.43	—	$8400\ \frac{(2.17-e_o)^2}{1+e_o}$	0.60	Kokusho and Esashi
Railroad ballast	0.68..0.78	—	$7230\ \frac{(2.97-e_o)^2}{1+e_o}$	0.38	Prange

Estimation of G_a from drained triaxial test

- G_a parameter should be equal to G_{ur}
- To estimate G_{ur} we have to know n and ν
- The default value of $\nu = \nu_{ur} = 0.2$ can be used while n can be taken from empirical formuli (usually $n = 0.5$).



Estimation of G_a from drained triaxial test



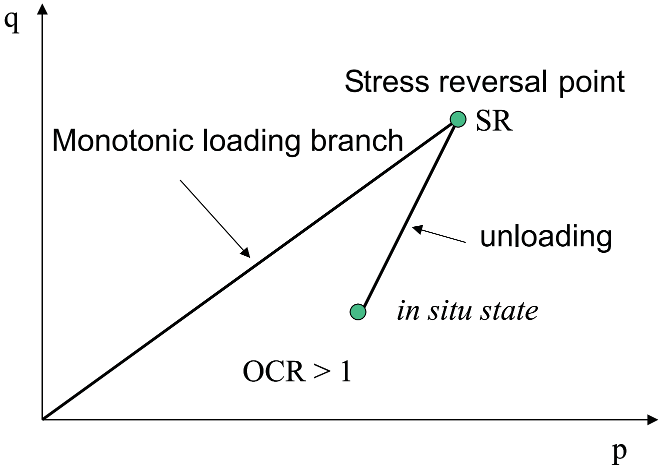
$$G_a \left(\frac{\sigma_3 + \frac{q_{un}}{3}}{p_{\text{ref}}} \right)^n = \frac{E_{ur}}{2\ (1 + \nu_{ur})} \quad \rightarrow \quad G_a = \frac{\frac{E_{ur}}{2\ (1+\nu_{ur})}}{\left(\frac{\sigma_3 + \frac{q_{un}}{3}}{p_{\text{ref}}} \right)^n}$$

NB. If we have two values of E_{ur} for two σ_3 values we may estimate also n parameter

Estimation of a_m parameter

- Once all parameters of the model, except a_m, are fixed one may simulate a single-element drained strain controlled triaxial test under constant confining stress $\sigma_3 = \text{const}$ for few assumed a_m values to fit demanded ratio $\frac{E_{50}}{E_{ur}}$.
- In case when ratio $\frac{E_{50}}{E_{ur}}$ is not known from the experiment the default value can be taken as $\frac{E_{50}}{E_{ur}} = 0.33$
- Larger values of a_m parameter yield softer response in $q - \varepsilon_1$ curves

We need to estimate p_c and r_1 parameters with aid of K_o^{SR} and OCR plus given $\boldsymbol{\sigma}_o$ state



Simulation of oedometer test

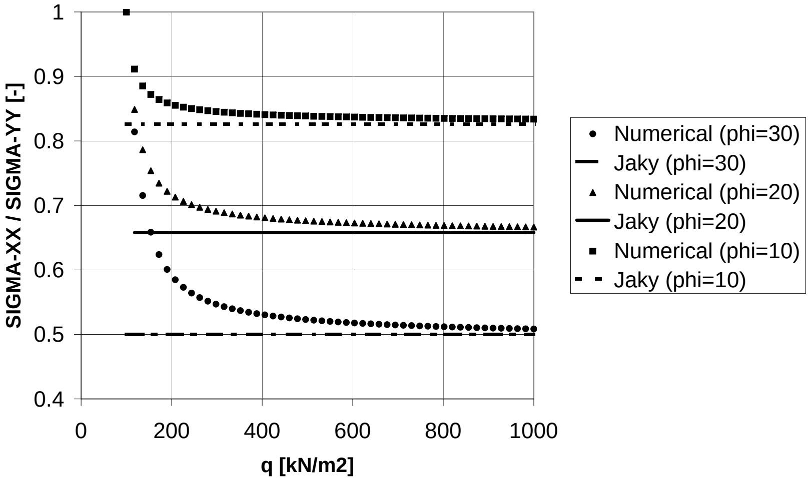


Figure: Variation of $\frac{\sigma_x}{\sigma_y}$ ratio during oedometric compression

Simulation of triaxial drained compression test (OCR=1)

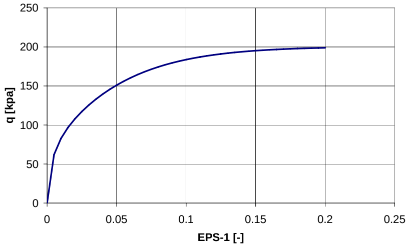


Figure: Shear characteristic in the triaxial compression test (OCR=1)

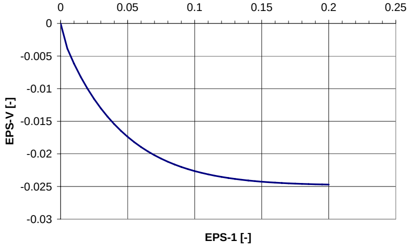


Figure: Dilatancy characteristic in the triaxial compression test (OCR=1)

Simulation of triaxial drained compression test (OCR=5)

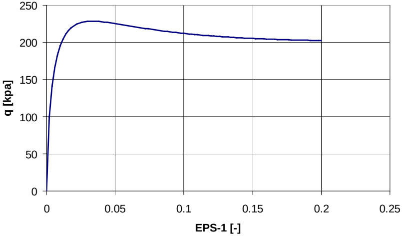


Figure: Shear characteristic in the triaxial compression test (OCR=5)

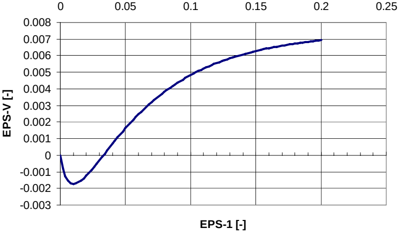


Figure: Dilatancy characteristic in the triaxial compression test (OCR=5)