

# Two-phase media

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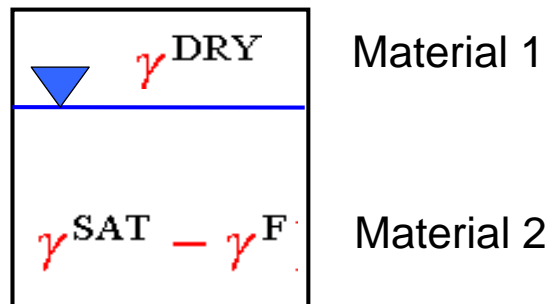
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## Effective stress analysis in **Deformation** mode: Simplified treatment of water

- Equilibrium:  $\sigma'_{ij,j} + f_i = 0$
- A different treatment of the gravity is necessary above and below water table
- $f_i = \gamma^{\text{DRY}} b_i$  above water table
- $f_i = (\gamma^{\text{SAT}} - \gamma^{\text{F}}) b_i$  under water table
- Hence two material zones must be distinguished



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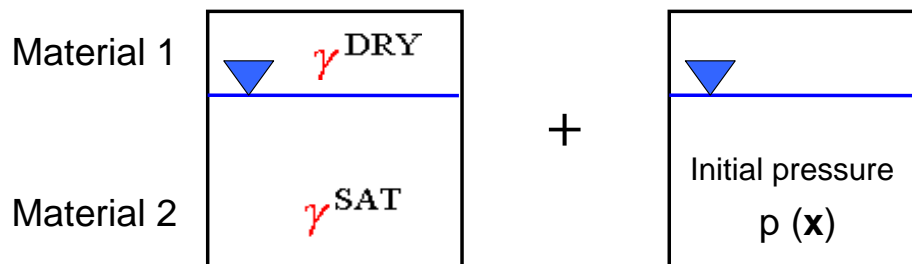
For simple cases with the horizontal phreatic surface one may still work under *Deformation* mode to take into account presence of the water (in this case a hydrostatic pressure is generated and positions of fluid particles remain unchanged). The first approach that can be used is called effective stress. In this case we have to distinguish between soil unit weights above and under water table. Note that the resulting stress state will be effective.

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## Total stress analysis in **Deformation** mode: Simplified treatment of water

- Equilibrium:  $(\sigma'_{ij} + \delta_{ij} p)_{,j} + f_i = 0$
- A different treatment of the gravity is necessary above and below water table
- $f_i = \gamma^{\text{DRY}} b_i$                       above water table
- $f_i = \gamma^{\text{SAT}} b_i$                       under water table
- Hence again two material zones must be distinguished



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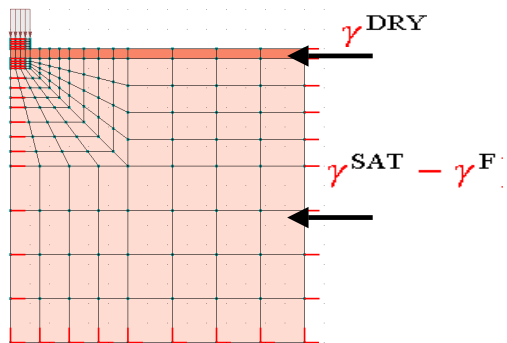
The second possibility that is fully compatible with the previous one is called total stress analysis. Here we have to distinguish between soil unit weights above and under the water table and in addition we have set the initial pore pressures (in preprocessor we can activate at the FE level *Initial conditions/Initial pressure*).

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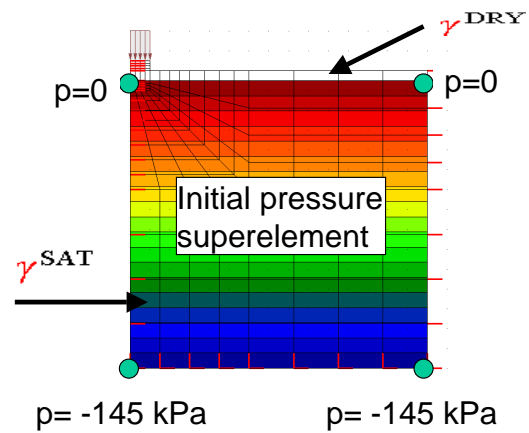


# Effective/total stress analyses in **Deformation** mode: Examples

**Effective stress:** footes.inp



**Total stress:** footwt.inp



**video**

- Remark: Both approaches are equivalent

These two examples show the application of both approaches.



## Coupled total stress analysis (CTS) in **Deformation+Flow** mode

Governing equations for two-phase fully or partially saturated medium

- Overall equilibrium:  $\sigma_{ij,j}^{\text{tot}} + f_i = 0$
- Effective stress principle by Bishop:  $\sigma_{ij}^{\text{tot}} = \sigma'_{ij} + S p \delta_{ij}$
- Fluid flow continuity:  $S \dot{\varepsilon}_{kk} + v_{k,k}^F - c \dot{p} = 0$
- Body load:  $f_i = (\gamma^{\text{DRY}} + S n \gamma^F) b_i$

Remark

The initial void ratio  $e_o$  (to be set at the material level) plays an important role especially in the CTS analyses

Coupled total stress analysis can be run exclusively under *Deformation+Flow* mode and it is equivalent to the *Consolidation* driver. This driver takes into account the effect of increase/decrease of pore pressure and corresponding deformation due to applied loading and similar effects due to variation of pore pressure BC in time. It is important that soil models must be set up based on effective strength parameters. All the effects like undrained shear strength etc.. will easily be reproduced by consolidation driver. This driver should be used for analyses of deep excavations to get proper assessment of sectional forces in diaphragm walls etc.. Relative time scale can be taken into account here. This approach requires a specific setting of soil unit weight. In the data we have to define unit soil weight but in its dry state  $\gamma_d$  because the real unit weight will depend on the value of current saturation ratio that is not explicitly known.



## Uncoupled total stress analysis (UTS) in Deformation+Flow mode

Governing equations for two-phase fully or partially saturated medium

- Overall equilibrium:  $\sigma_{ij,j}^{\text{tot}} + f_i = 0$
- Effective stress principle by Bishop:  $\sigma_{ij}^{\text{tot}} = \sigma'_{ij} + S p \delta_{ij}$
- Fluid flow continuity (steady state):  $v_{k,k}^F = 0$
- Body load:  $f_i = (\gamma^{\text{DRY}} + S n \gamma^F) b_i$
- In this case each computational step consists of two:
  - 1 Steady state fluid flow; it yields current pressure state  $p(t_{N+1})$
  - 2 Evaluation of mechanical state  $(u_{N+1}, \sigma_{N+1})$  with the explicit pore pressures state  $p(t_{N+1})$

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Running uncoupled total stress analysis we can couple deformation and flow only in one direction. It means that the variation of the pressure field may cause deformations but not vice versa. Setting unit weight of soil is the same as for CTS approach.

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## UTS/CTS analysis: remarks on setting soil strength parameters

- All constitutive models are formulated in terms of effective stresses ( $\Delta\sigma' = \mathbf{D}^{\text{ep}}\Delta\varepsilon$ )
- Hence effective strength parameters must be used ( $\phi', c'$ )
- Note that theoretically for fully saturated soils  $c' \rightarrow 0$  (!)
- Example: for tertiary soils we can find in norms that  $\phi = 15^\circ$  and  $c = 40$  kPa, but these are not effective parameters. From drained triaxial test we will get for this kind of soils  $\phi' = 27^\circ$  and  $c' = 7$  kPa
- Is there a mistake here ? No, this effect can easily be reproduced when coupling constitutive soil model based on effective parameters and Darcy flow by means of CTS analysis

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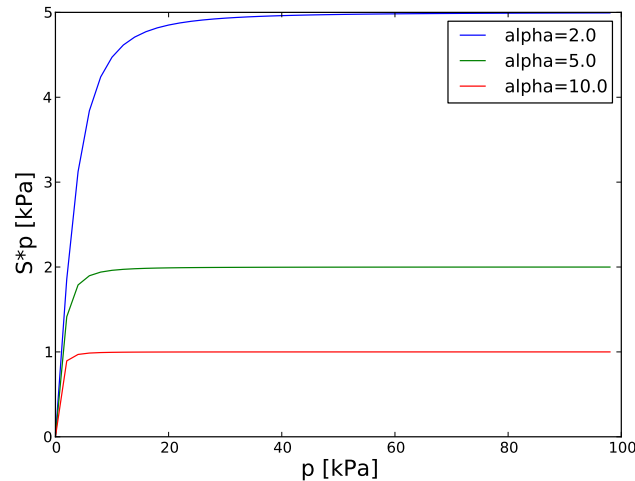
The effective strength parameters ( $\phi'$  and  $c'$ ) are usually obtained from drained triaxial testing. In the real experiments we always get some residual cohesion  $c'$  of order of few kPa (2 to 5 kPa) due to incomplete saturation ( some voids are closed with gas bubbles ).

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## UTS/CTS analysis: effect of $\alpha$ and $S_r$ parameters on soil strength

- Effective stress principle by Bishop:  $\sigma_{ij}^{\text{tot}} = \sigma'_{ij} + S p \delta_{ij}$
- Let us find a limit for  $S p$  (when  $p \rightarrow \infty$ ) using van Genuchten's law
- For  $S_r = 0.0$  and  $p \rightarrow \infty$   $S p \rightarrow \frac{\gamma^F}{\alpha}$



$$S_r = 0.0$$

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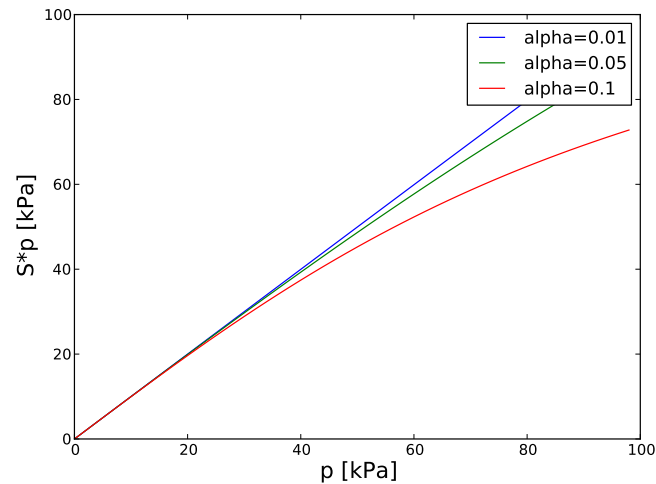
When running UTS/CTS analyses we cannot forget about effect of extra cohesion that is produced by the theory in the zone of partial saturation. By assuming  $S_r = 0.0$  we will be always on the safe side of the safety estimation. In the worst case the extra cohesion will not exceed  $\frac{\gamma^F}{\alpha}$

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## UTS/CTS analysis: effect of $\alpha$ and $S_r$ parameters on soil strength

- For  $S_r > 0.0$  and  $p \rightarrow \infty$   $S p \rightarrow \infty$



$$S_r = 0.10$$

Conclusion: To avoid too much of extra cohesion set  $S_r = 0$  and  $\alpha$  as for given soil (for instance when  $\alpha = 1$  the extra cohesion will not exceed 10 kPa)

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For low values of  $\alpha$  and nonzero  $S_r$  the extra cohesion can be large. This may lead to substantial overestimation of limit loads or safety factors.

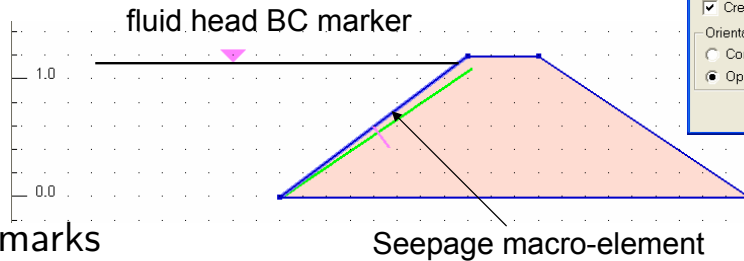
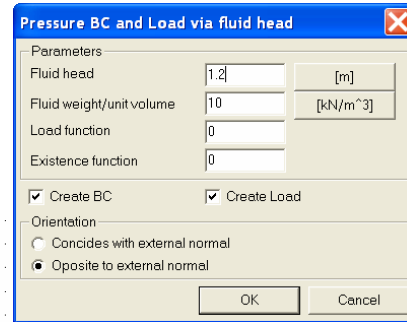
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# UTS/CTS analysis: BC on submerged boundaries

Meaning of colors of fluid head BC markers:

	(magenta) : BC + Load
	(blue) : BC only
	(red) : Load only



## Remarks

- 1 Imposed pressure head will be applied to all nodes on the boundary designed by selected faces if the checkbox  **Create BC** is set to ON
- 2 Equivalent pressure load will be applied to the boundary once the checkbox  **Create Load** is set to ON
- 3 The load time function applies to the pressure head value  $h$
- 4 Mesh refinement is automatically handled and no extra action from the user is needed
- 5 Seepage surface elements must be generated at the boundary where Fluid head BC is applied (this allows to vary the water table in time)

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In the two-phase applications we neglect presence of the water (itself) above submerged boundaries and we do not discretize it. This is so because we assume that water has no shear stiffness and hence its presence can be replaced by an equivalent loading. By following this assumption we have to apply surface loading on submerged boundaries. This definition is automatic when pressure BC on submerged boundaries is made with aid of fluid head.

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## UTS/CTS analysis: Drivers

- ① Drivers give an instruction to the program what is to be computed and in which order
- ② By default the **Initial State** driver is set up as the first and the only one in the list for execution (it always yields zero deformation !)
- ③ Available drivers:
  - **Initial state**
  - **Stability**
  - **Time Dependent**
    - **Driven Load** is equivalent to uncoupled analysis
    - **Consolidation** is equivalent to coupled analysis
  - **Dynamics**
    - **Driven Load** is equivalent to uncoupled time history analysis
    - **Consolidation** is equivalent to coupled time history analysis

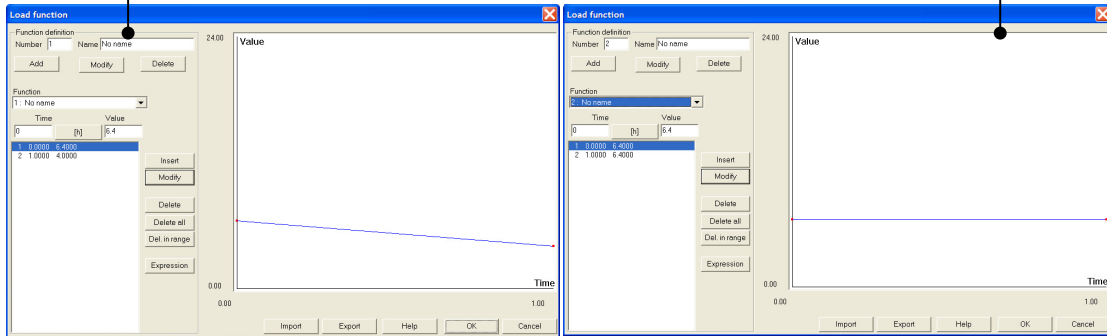
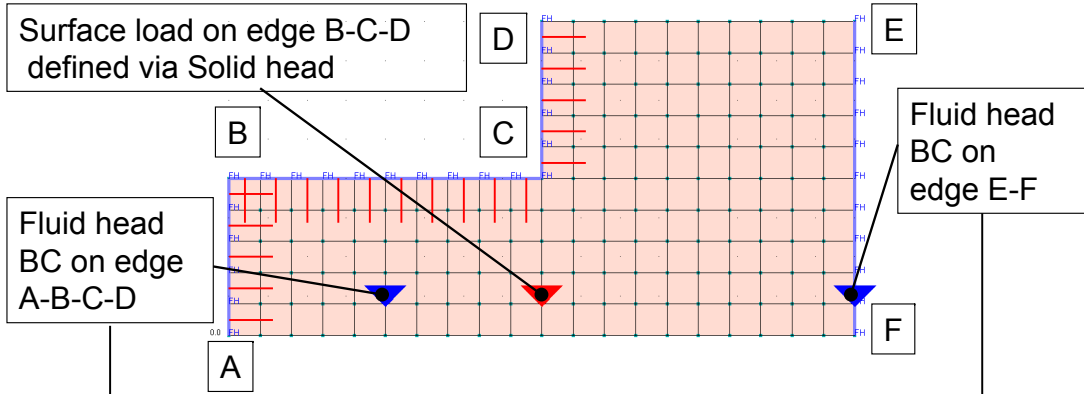
NB. The **Initial state** is equivalent to uncoupled analysis with the steady state fluid flow solved at time  $t = 0$

The coupled (*Consolidation*) and uncoupled (*Driven Load*) analyses can be run for both static and dynamic problems.



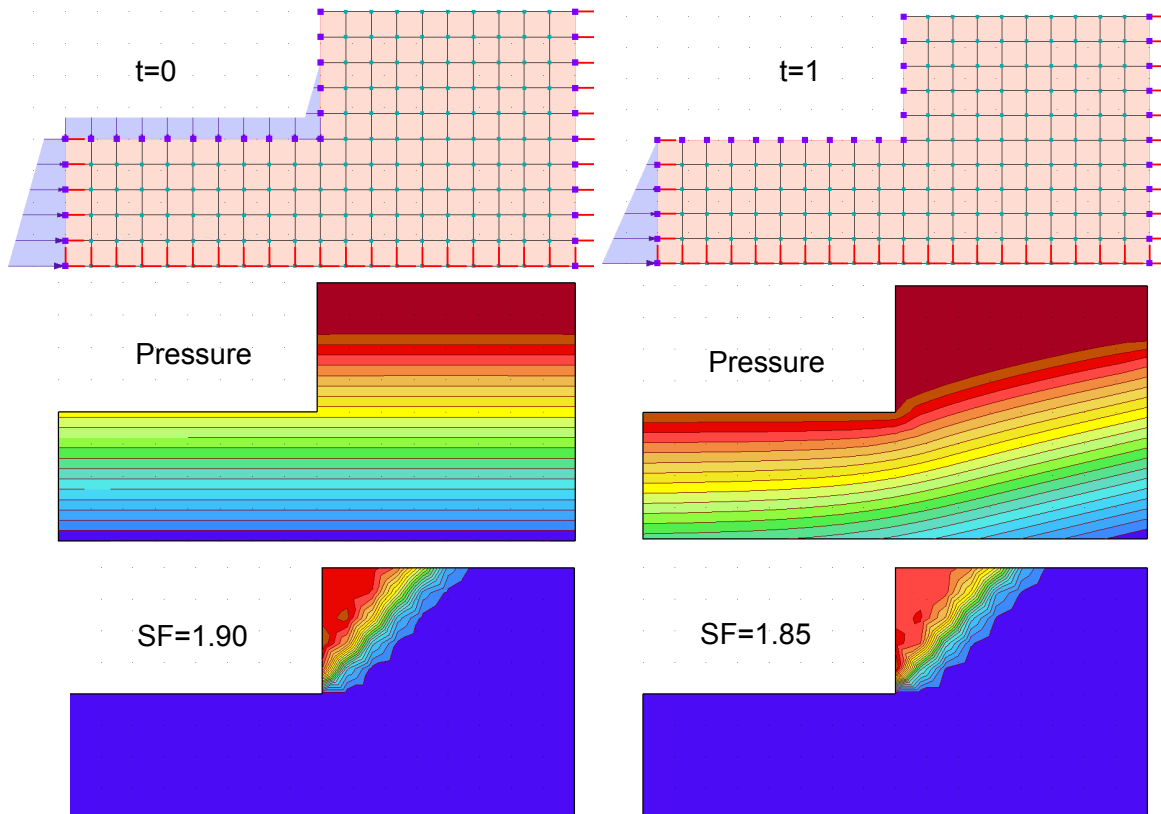
# UTS analysis: Ex. cut slope stability: Data

Input file : cuttrbc.inp



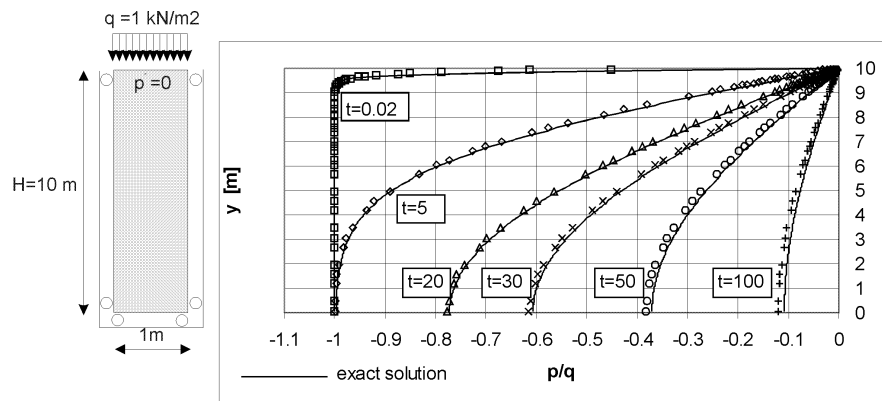
# UTS analysis: Ex. cut slope stability: Results

Input file : cuttrbc.inp





# CTS analysis: Consolidation (Terzaghi problem)



## Analytical solution:

$$\frac{p^F(z, t)}{q} = \sum_{m=0}^{\infty} \left( \frac{2}{M} \right) \sin \left( \frac{M \cdot z}{H} \right) \exp(-M^2 \cdot T_v)$$

$$M = \frac{\pi(2m+1)}{2}$$

$$T_v = \left( \frac{C_v \cdot t}{H^2} \right); \quad C_v = \left( \frac{E_{oed} \cdot k}{\gamma^F} \right).$$

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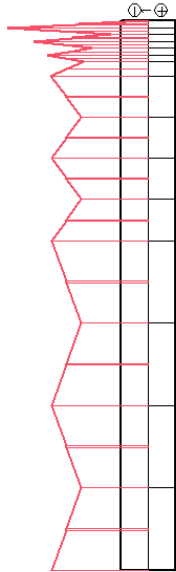
To model Terzaghi problem we have to run the CTS analysis with the two drivers, first the *Initial State* and then *Consolidation*. The first one generates the initial effective stresses and pore pressures. The second will generate excess pore pressure due to applied loading at the top. To analyze excess pore pressure we can use postprocessor and to activate visualization of difference of results between the two selected time instances. If we select the first solution from the initial state driver as a reference one then we can easily trace evolution of the excess pore pressure

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# CTS analysis: Problems of pressure oscillations

Pressure profile at  
 $t = 10^{-4}$  d



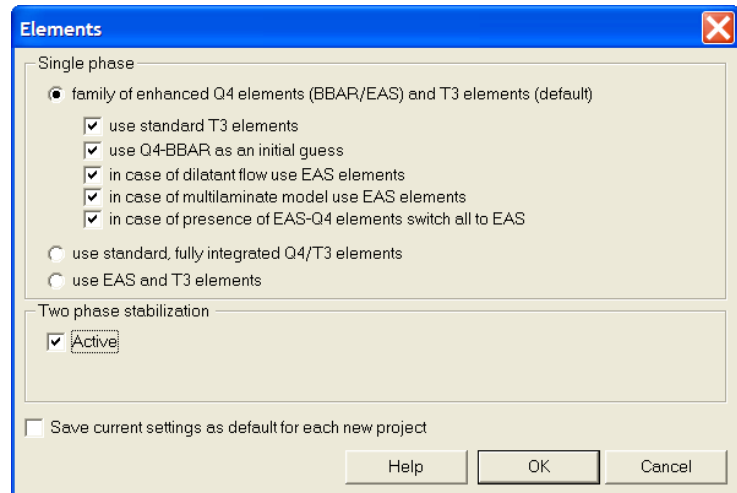
- Time step must be greater than certain value to avoid spurious pressure oscillations (Vermeer 1981)

$$\Delta t_{crit} \geq \frac{1}{6} \cdot \frac{h^2}{\theta C_v}$$

- In Z\_SOIL this condition takes the form

$$\Delta t_{crit} \geq \frac{1}{4} \cdot \frac{h^2}{\theta C_v}$$

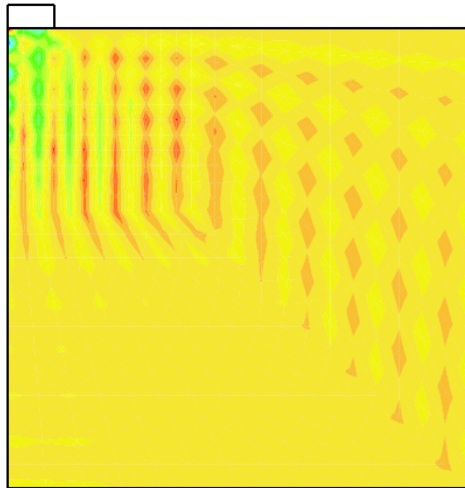
- To cancel pressure oscillations activate 2-phase stabilization



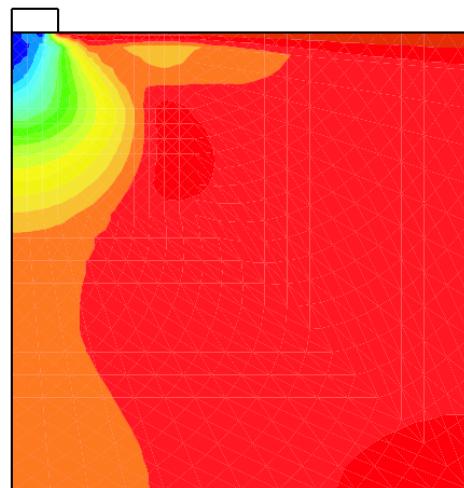
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Presence of pressure oscillations may appear when very short time steps (smaller than  $\Delta t_{crit}$ ) are used. These spatial pressure oscillations may generate certain convergence problems if contact interfaces remain in these zones. If we observe such an effect it is possible to suppress it by activating *Two-phase stabilization* option under menu *Control/Finite elements* in advanced mode.

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Nostabilized pressure solution



Stabilized pressure solution

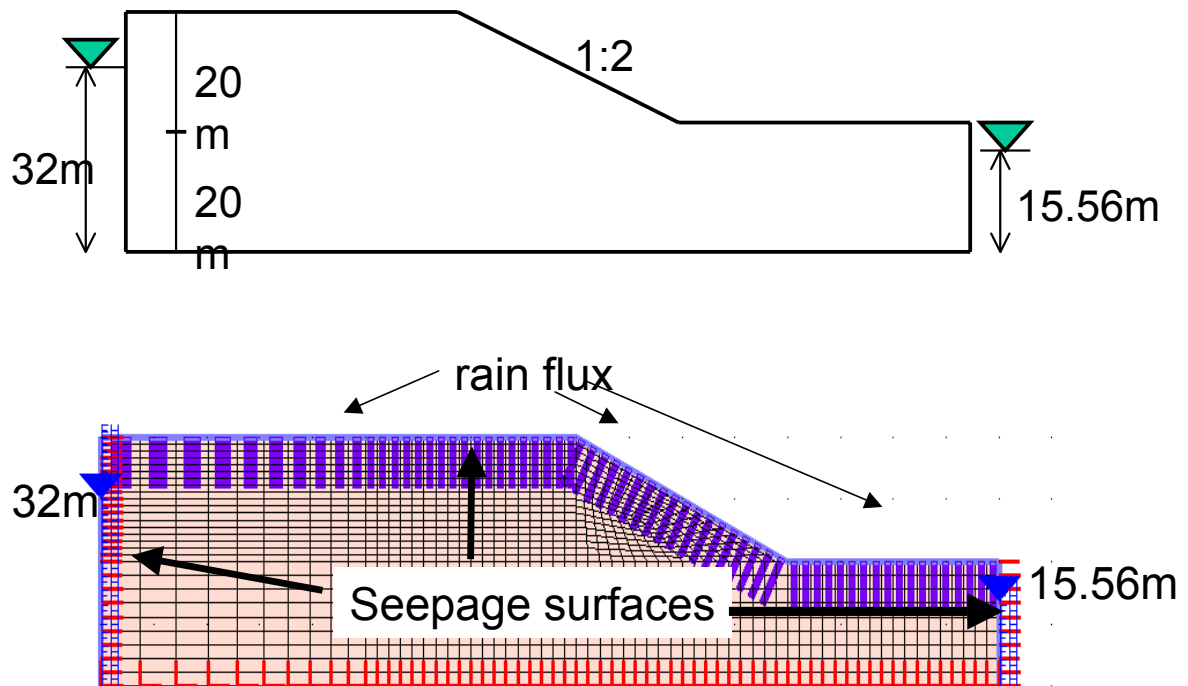
## Remarks

- With this option quasi-undrained analysis run with consolidation drivers is possible
- In case when larger time steps are to be used switch-off this option as it increases computation time

The two color contours show the effect of pressure stabilization.



## CTS analysis: Stability of slopes subject to rain flux



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As the pressure BCs are kept constant on left and right boundaries model must be elongated to eliminate the effect of boundary conditions on final result. At the top of the slope we apply fluid flux with its evolution in time driven by a load time function and we add seepage surface elements to prevent rain inflow once the slope is fully saturated. The rain flux is limited by the value of permeability coefficient.

Elongating of the model would not be needed once the pressure BCs on left and right boundaries were cancelled when running consolidation driver (in this case these boundaries would remain fully impermeable).

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# CTS analysis: Stability of slopes subject to rain flux

## Drivers

**Analysis and Drivers definition**

Drivers definition

Driver: Initial State

Ini. load fac. 0.5 | Fin. load fac. 1 | Increment 0.1

Nonl. solver settings: Default

Set

N°	Driver		Ini. load factor	Fin. load fact...	Increment		Nonl. solver settin...
1	Initial State		0.5000	1.0000	0.1000		Default
2	Time Dependent	Consolidation	0	10.0000	0.5000	1.0000	Default
3	Stability	tg(phi),c	1.0000	2.0000	0.0100		Default
4	Time Dependent	Consolidation	10.0000	20.0000	0.5000	1.0000	Default
5	Stability	tg(phi),c	1.0000	2.0000	0.0100		Default
6	Time Dependent	Consolidation	20.0000	30.0000	0.5000	1.0000	Default
7	Stability	tg(phi),c	1.0000	2.0000	0.0100		Default
8	Time Dependent	Consolidation	30.0000	40.0000	0.5000	1.0000	Default
9	Stability	tg(phi),c	1.0000	2.0000	0.0100		Default
10	Time Dependent	Consolidation	40.0000	200.0000	0.5000	1.0000	Default

Add | Insert | Delete | Modify

Analysis & Problem type

Analysis: Plane Strain

Problem: Deformation + Flow

Associated preprocessed projects:

Heat project:  Browse...

Humidity project:  Browse...

OK | Cancel | Help

Advanced



# CTS analysis: Stability of slopes subject to rain flux

## Flux definition

**Load function**

Function definition

Number 1 | Name No name

Add | Modify | Delete

Function: 1: No name

Time [h] | Value

1	1.0000	0.0000
2	2.0000	0.0100
3	10.0000	0.0100
4	12.0000	0.0200
5	16.0000	0.0200
6	20.0000	0.0200
7	22.0000	0.0300
8	24.0000	0.0300
9	30.0000	0.0300
10	40.0000	0.0400
11	80.0000	0.0400

Insert | Modify | Delete | Delete all | Del. in range | Expression

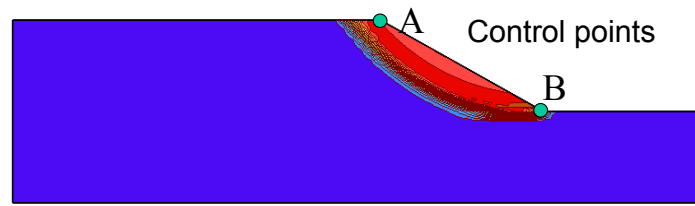
Import | Export | Help | OK | Cancel

NB. Flux must be smaller than seepage coefficient

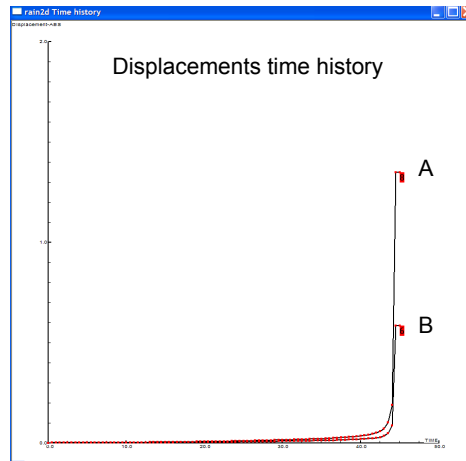


# CTS analysis: Stability of slopes subject to rain flux

Failure mode



Displ. time history



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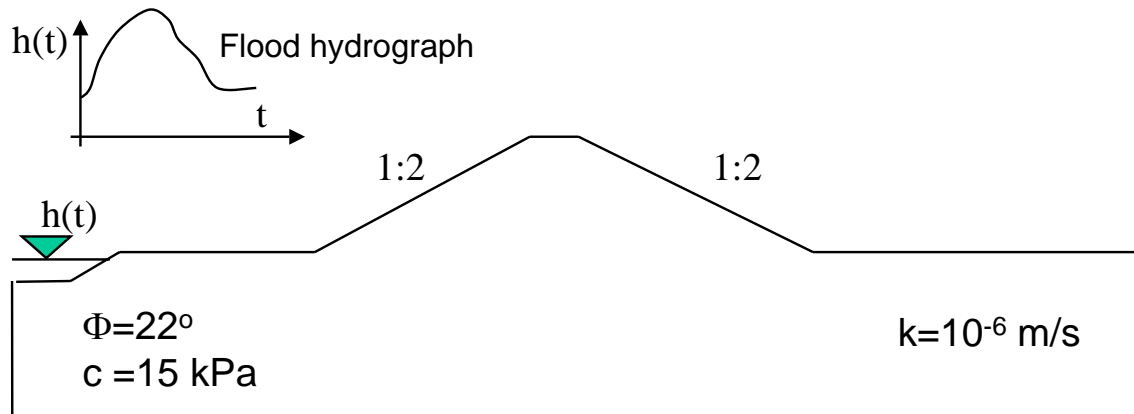
These results show that loss of stability due to long periods of raining can nicely be reproduced by ZSOIL. Note that loss of stability is sudden.

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## CTS analysis: Stability of embankments subject to flood

Problem: find minimum value of the safety factor and failure mode during flood transition for a given hydrograph



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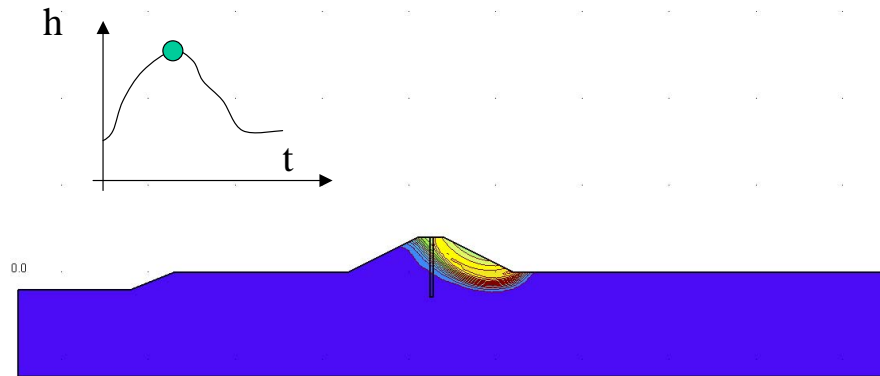
To analyze flood transition the CTS approach must be used. The pressure BCs on the internal face of the dike are defined with aid of fluid head BC + seepage elements that are extra added along that surface. On the external face seepage elements are added too to allow for free water outflow once the phreatic surface hits the external face.

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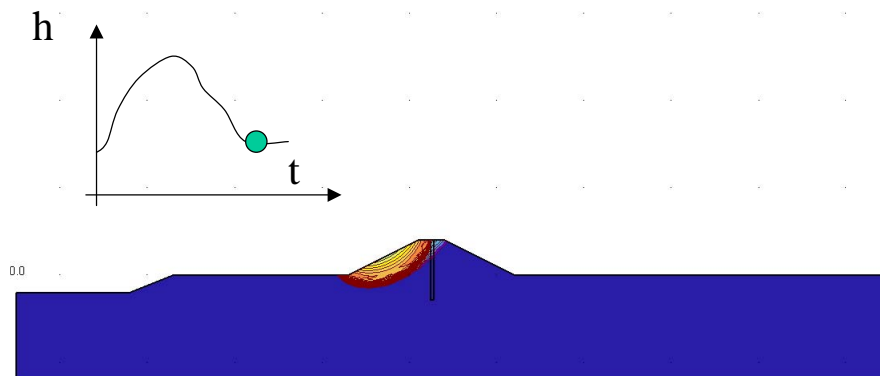


# CTS analysis: Stability of embankments subject to flood

Failure mode at the maximum water elevation



Failure mode just after flood transition



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This example shows that do not need to make any extra assumptions to compute safety coefficients at any stage of flood transition. All basic features are embedded in the theory implemented in ZSOIL. Note that the two different failure mechanisms are captured. When running consolidation driver one may activate safety analysis at any time instance in one run.

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