

Seepage

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Seepage analysis for fully/partially saturated media

1 Steady state

$$v_{k,k}^F = 0$$

2 Transient

$$v_{k,k}^F - c \dot{p} = 0$$

• Darcy velocity

$$v_i^F = -k_{ij} k_r(S) \left(-\frac{p}{\gamma^F} + z \right)_{,j}$$

• $k_r(S)$ function

$$k_r = \frac{(S - S_r)^3}{(1 - S_r)^3}$$

• $S(p)$ (van Genuchten)

$$S(p) = S_r + \frac{1 - S_r}{\left[1 + \left(\alpha \frac{p}{\gamma^F} \right)^2 \right]^{1/2}}$$

• $c(p)$ storage function

$$c = c(p) = n \left(\frac{S}{K_F} + \frac{dS}{dp} \right)$$

NB. Saturation ratio is denoted by S , porosity by n , fluid bulk modulus by K_F , residual saturation ratio by S_r fluid unit weight by γ^F

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If we would assume that voids in soil are filled with an incompressible fluid then the differential equation $v_{k,k}^F = 0$ would mean that the fluid flux that enters the control volume in a time unit must quit this volume at the same time. Fluid compressibility and initial void ratio neither do not affect steady state solution.

In the transient case we have to take into account the effect of fluid compressibility that is influenced by the fact that the fluid is compressible itself and soil can be partially saturated. Hence in the storage function $c(p)$ we can notice these two effects.

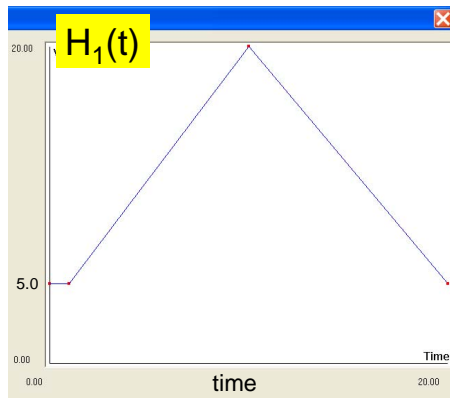
As we want to solve seepage problems on fixed meshes (that do not adapt to the position of free surface) we have to modify the Darcy flow rule assuming that permeability in the zone of partial saturation decreases with the saturation ratio S while the saturation ratio is a vanishing function of a suction pressure. This function $S(p)$ is strongly influenced by parameter α and is represented by van Genuchten's law.

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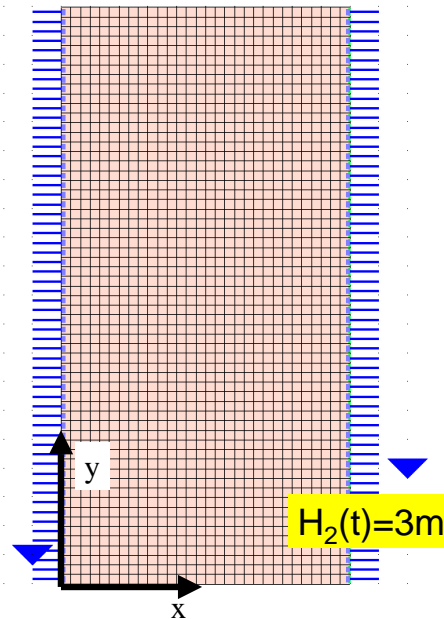


Steady state vs transient seepage analysis

Let us consider a simple example of a soil layer subject to varying water elevation on the upstream side.



Varying water elevation $H_1(t)$



- **Steady state solution** [video](#)
- **Transient solution** [video](#)

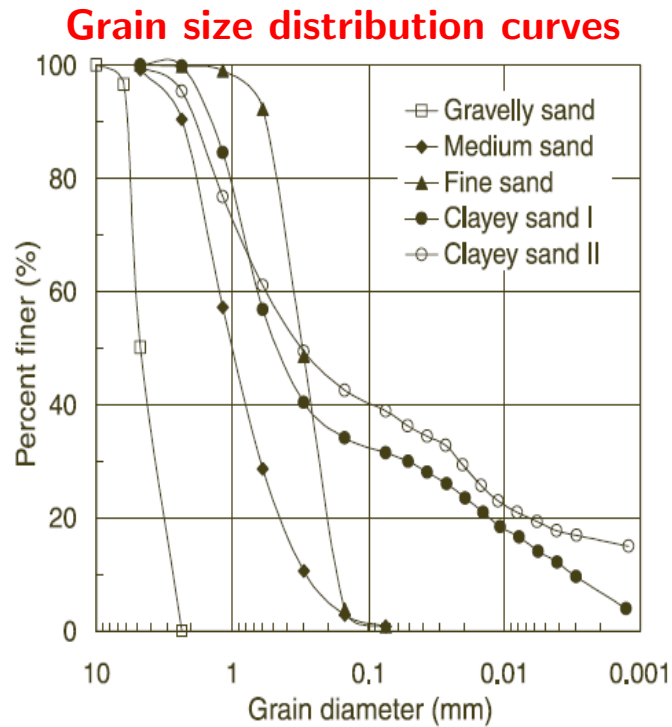
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In this example we will try to model filling and then sudden drawdown of the water level in a reservoir. Let us assume that the mesh represents a clayey core that should reduce the outflow from reservoir. We will consider two cases i.e. steady state one and the transient one. In the steady state the time parameter is an artificial one that allows to perform multi-steady state analysis for a sequence of set of pressure boundary conditions. In the transient case it is absolutely a real one. Transient case requires time stepping.

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Partial saturation: experimental data (after Yang*)



* Yang et al.: Factors affecting drying and wetting soil-water characteristic curves of sandy soils. Can. Geotech. J. 41, p. 908-920, 2004.

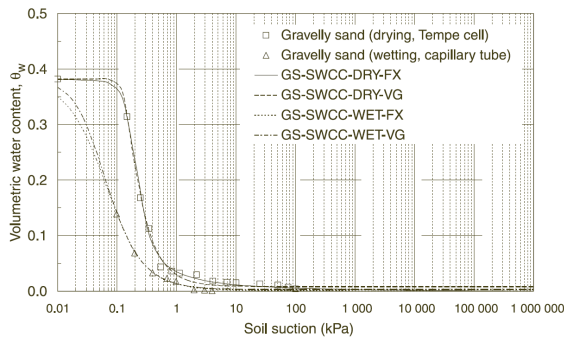
In the following slides we will see results of certain experiments performed by Yang et.al. to verify van Genuchten's law. Five different soils are tested and grain size distribution curves are shown in the figure. Other material properties of these soils are given in the following table.

	Gravelly sand	Medium sand	Fine sand	Clayey sand I	Clayey sand II
Unified Soil Classification System	SP	SP	SP	SC	SC
Specific gravity, G_s	2.62	2.60	2.65	2.64	2.59
Grain-size analysis results					
D_{60} (mm)	5.15	1.25	0.35	0.66	0.56
D_{30} (mm)	3.68	0.62	0.23	0.051	0.021
D_{10} (mm)	2.73	0.29	0.17	0.003	0.0005
Coefficient of uniformity, C_u	1.89	4.31	2.06	220	1120
Coefficient of curvature, C_c	0.96	1.06	0.89	1.31	1.58
Gravel content (larger than 4.75 mm; %)	49.9	0.8	0	0	0.2
Fines content (finer than 0.075 mm; %)	0	0.8	0.8	31.5	38.9
Atterberg limits (on minus 0.425 mm sieve fraction)					
Liquid limit, LL	—	—	—	31	48
Plastic limit, PL	—	—	—	21	27
Plasticity index, PI	—	—	—	10	21
Soil properties used in SWCC test					
Dry density of soil, ρ_d (Mg/m^3)	1.62	1.69	1.56	1.72	1.47
Void ratio, e	0.617	0.538	0.699	0.535	0.762
Porosity (%)	38.2	35.0	41.1	34.8	43.2

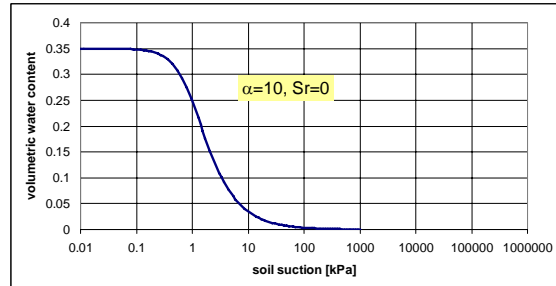
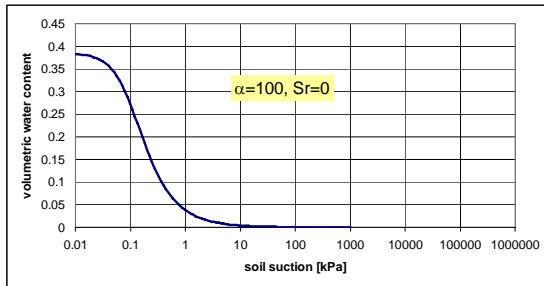
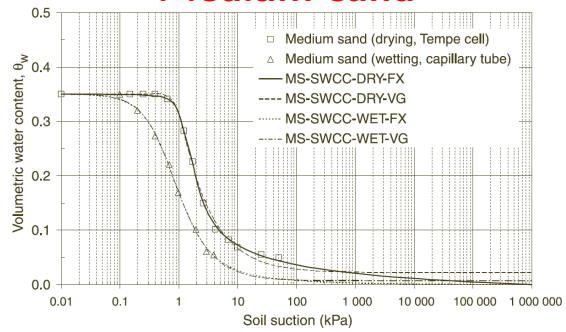


Partial saturation: experimental data (after Yang*)

Gravelly sand



Medium sand

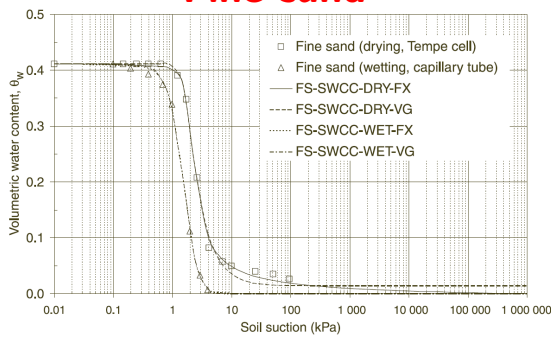


We can notice that for coarse materials value of α parameter may tend up to 100.0. However, numerical experience shows that in some situations setting α above 10.0 may cause divergence of the iterative process. We should notice that wetting/drying curves do not follow the same path in $\theta - p$ (θ is the water content). In the theory used in the code we neglect this hysteretic behavior and try to fit an averaged curve.

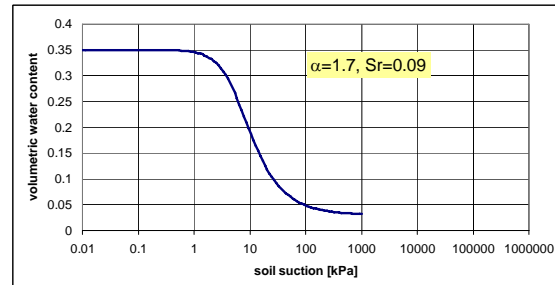
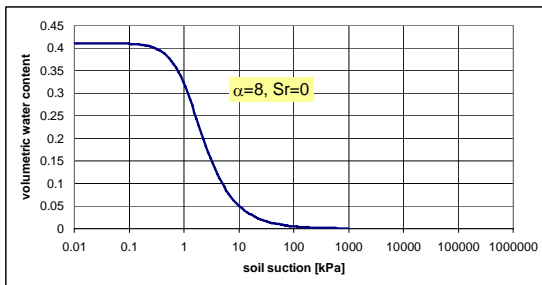
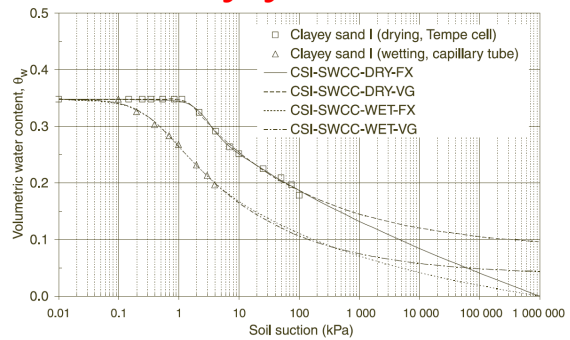


Partial saturation: experimental data (after Yang*)

Fine sand



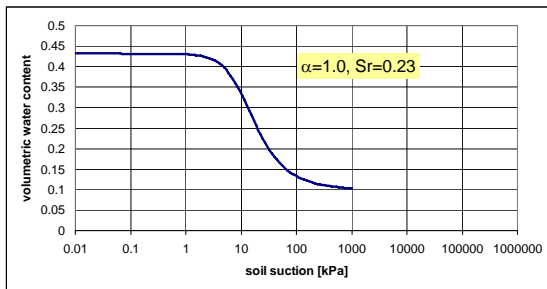
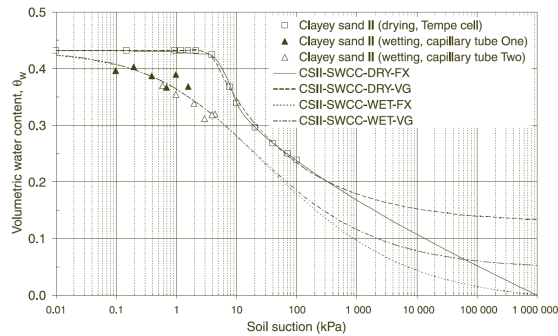
Clayey sand I



We can see that between fine sands (cohesionless material) and clayey sands (cohesive material) α may drop from value 8.0 to 1.7 (let say 2.0).



Clayey sand II



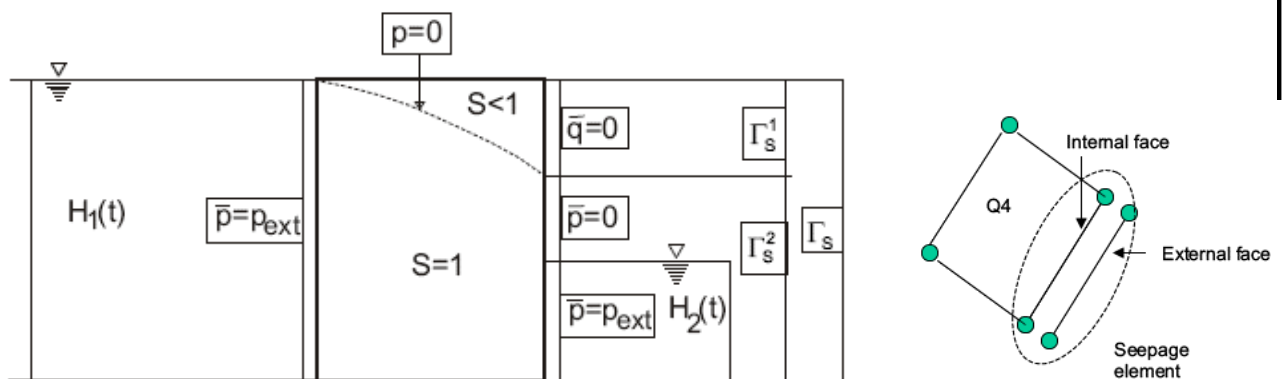
For clayey sands with majority of silt and clay particles $\alpha = 1.0$ and we observe that residual saturation ratio S_r becomes larger than 0.0.

For two-phase applications (stability, bearing capacity) we have to be careful with selection of $S_r > 0.0$ because of the effect of extra cohesion (this important point will be discussed later on in lecture devoted to two-phase problems)



Boundary conditions

- ① Imposed pressure at nodal points
reaction → flux (outflow) at the node
 - Can be defined directly by specifying pressure values
 - Can be defined using pressure head definition ($h = -\frac{p}{\gamma_F} + y$)
- ② Distributed fluid fluxes applied to edges(2D)/facets(3D)
reaction → pressures at nodes (be careful when plasticity is on for two-phase problems)
- ③ Special treatment is required along seepage surfaces



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Boundary conditions can be set by means of prescribed fluid fluxes (at given edge or face) or pressure at a given nodal point. For steady state problems we have to prescribe pressure value at least at one node. In case of few subdomains that are isolated in each of them we must fix pressure value at least in one node. If we do not do it computation will fail to converge and in the *.log file we will get info like *null pivot detected at node: xxxx*. In the transient case once the initial conditions are set we can deactivate all pressure BCs.

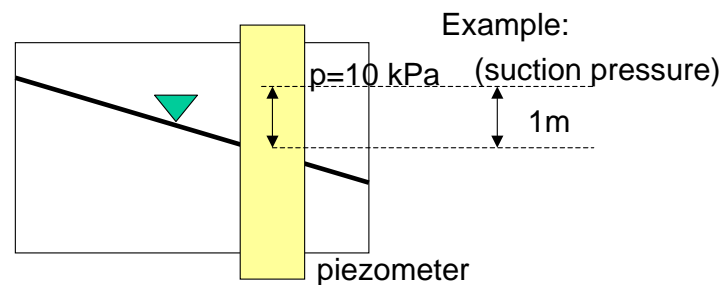
In the example of clay core we do not know a priori where is the intersection point of phreatic surface with the boundary Γ_s . This position is important because above the phreatic surface we would like to set boundary condition $q = 0$ (fluid flux equal to zero because of the impermeable boundary), while below, we would like to have $p = 0$ (fully permeable boundary) above downstream water level $H_2(t)$ and hydrostatic distribution of pressure under downstream water table. Note that lack of any pressure/flux BC means that all boundaries are fully impermeable.

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Seepage surface elements: remarks

- 1 Seepage surface elements automatically switch from pressure BC to fluid flux ($q=0$) or vice versa by looking inward and outward the domain (seepage surface element has 2 layers of nodes)
- 2 If the pressure BC is not prescribed on outer face of seepage surface then it is fixed to 0 by default
- 3 Positive pore pressure does not have a meaning of capillary pressure (!)



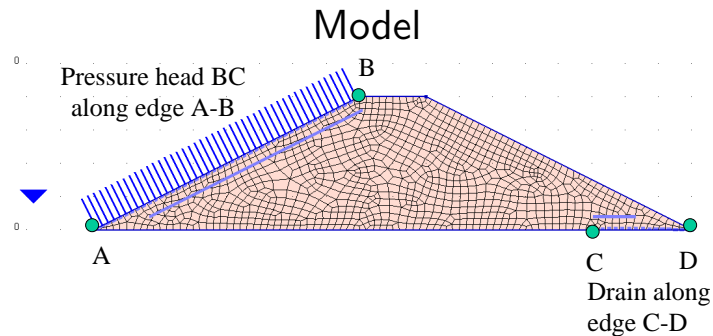
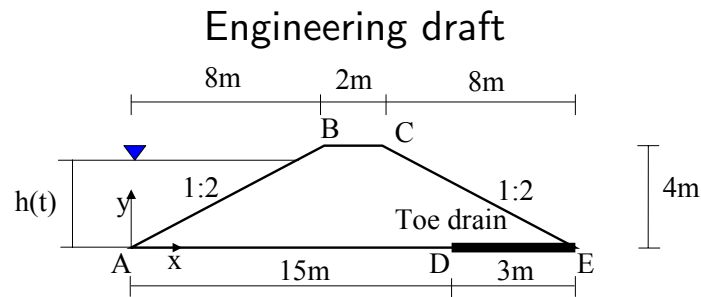
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This automatic switch from pressure to flux type BC that is generated by seepage surface elements is achieved with aid of penalty method. In this method we have to specify value of the penalty parameter k_v . This parameter is set internally in the code and in most situations its estimation is correct. However, one may always build an example (it is usually academic) that may show that this setting is not good enough. Hence at the material level user may scale our setting by a multiplier to k_v parameter. The default value of that multiplier is 1.0.

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Seepage elements: when should we use them ?



- Add seepage along A-B as water elevation varies in time and pressure BC is defined through pressure head
- Add seepage along C-D to model drain behaviour (with external pressure equal to zero)

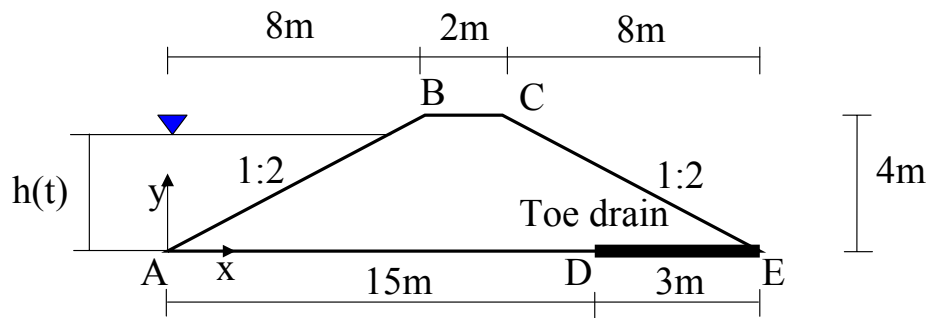
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Modeling drains can easily be made by using seepage surface elements (with zero external pressure BC (this is default setting if we do not do anything)). These seepage elements should be put on each boundary where we apply fluid head type BC. If in the solution we observe spurious compressive ($-$) pressures on (dry) free boundaries we should regenerate the model by adding seepage surface elements there. If in the above example compressive pressures would appear on the downstream face it means that the designed drain zone is too short.

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Pressure BC: definition through pressure head



- Definition of pressure head: $h(t) = -\frac{p(t)}{\gamma^F} + y$
- Benefit: by a single value we can set BC on whole edge A-B
- $h(t) = h_o \times LTF(t)$ (usually $h_o = 1$ m and $LTF(t)$ includes the information on variation of pressure head in time)
- **If we apply pressure head BC on some boundary we should apply it in conjunction with seepage surface elements**

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In order to handle varying pressure BC (in time) on the upstream face of the dike we have to add seepage elements there and to apply fluid head type BC.

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Drivers: General information.

- ① Drivers give an instruction to the program what is to be computed and in which order
- ② By default the **Initial State** driver is set up as the first and the only one in the list for execution
(it is equivalent to the steady state run at time $t = 0$)
- ③ Available drivers:
 - **Initial State**
 - **Time Dependent** **Steady State**
 - **Time Dependent** **Transient**

NB. **Time Dependent** **Transient** driver activates integration in time hence time stepping plays an important role

Sequence of drivers can be set under menu *Cotrol/Analysis & Drivers*. The *Initial State* for *Flow* mode is solved (if activated) once at $t=0$. The *Initial State* driver may serve as a generator of the initial condition for further transient analysis.



Initial conditions for transient case

Transient seepage requires to set the initial conditions (initial pressures)

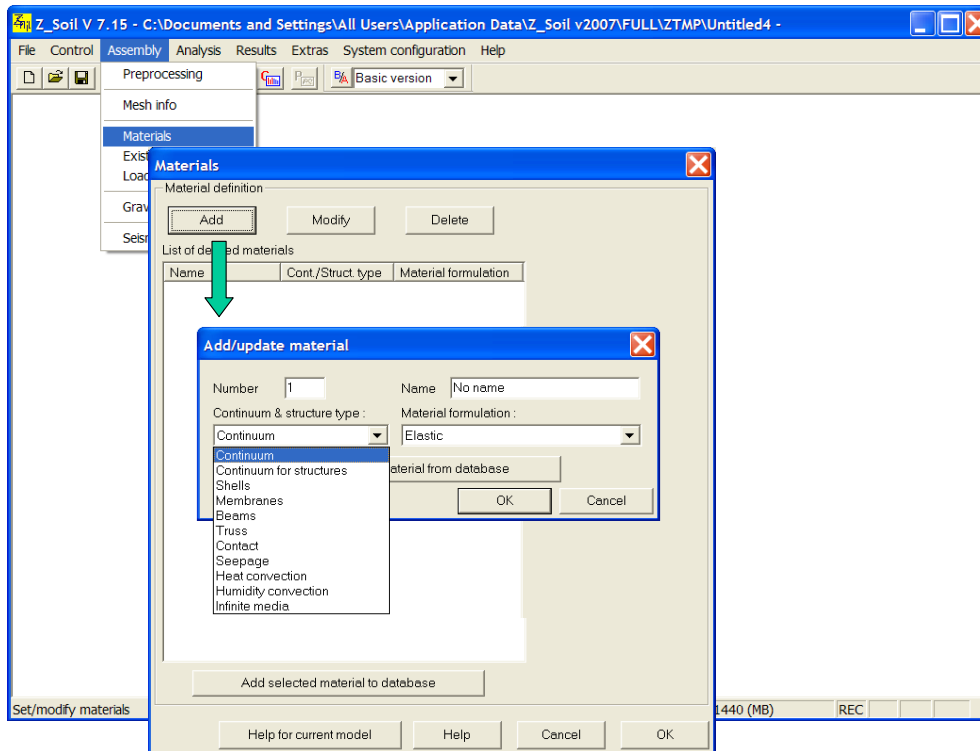
- ① The initial pressures can be defined through preliminary **Steady state** analysis (**Initial state=Steady state** at $t=0$), in this case the minimum set of drivers consists of
 - **Initial State driver**
 - **Time Dependent** → **Steady State**
- ② The initial pressures can directly be set in the preprocessor using **FE model/Initial conditions/Initial pressure** option, in this case the minimum set of drivers consists of
 - **Time dependent** → **Transient**

When we start the transient driver all existing BCs can be deactivated through the existence function attribute attached to to the given pressure boundary condition (nodal prescribed pressure or fluid head).

To start directly the transient analysis (without Initial State or preliminary Steady State drivers) one has to set the meaningful initial pressure field. This can be made with aid of pressure super-elements (regions with 4 vertices in 2D and 8 vertices (brick like) in 3D) in the preprocessing stage at the FE model level (*Initial conditions/Initial pressure*).



Structure types/Material formulations



NB. Any model can be applied for seepage analysis

To analyze flow problems one may use any type of the solid constitutive model designed for continuum (default is Elastic) All models, in terms of the seepage, will produce same results.



Material data: Unit Weight group

Unit weights /masses

Unit weights/masses

Weight / unit volume γ 19 [kN/m³]

Mass / unit volume ρ 1937.46080 [kg/m³]

Multiplier for body forces 1

Multiplier for mass matrix 1

Fluid weight / unit volume γ_f 10 [kN/m³]

Fluid mass / unit volume ρ_f 1019.71621 [kg/m³]

Initial void ratio e_o 0

Global gravity direction

N.B.
for deformation (single phase) analysis in total stresses specify the saturated specific weight γ_{SAT} below the water table and the dry specific weight γ_D above;
in effective stress analysis specify buoyant weight $\gamma_B = \gamma_{SAT} - \gamma_w$;
for deformation+flow (two-phase) specify γ_D (dry) and initial void ratio e_o ;
 γ will be computed as $\gamma = \gamma_D + n S \gamma_w$; $n = e_o / (1 + e_o)$; S - saturation ratio.

Data mode Standard

OK
Cancel
Help

- e_o plays crucial role in transient seepage analyses hence must be set to a meaningful value

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- $e_o = \frac{n}{1 - n}$, $n = 1 - \frac{\gamma_d}{\gamma_s}$, $\gamma_d = \frac{\gamma}{1 + w}$
- w is the humidity
- γ_s is a solid specific weight (usually $\gamma_s = 26.5 \text{ kN/m}^3$)
- γ is an unit soil weight for a given humidity
- If we know soil unit weight at stage of full saturation γ_{sat} then $\gamma_d = \gamma_{sat} - n \gamma^F$

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Material data: Flow group for 2D problems

2D Flow [X]

Fluid bulk modulus β_F [kN/m²]

Darcy's coefficient

Darcy's coefficient K_x [m/day]

Darcy's coefficient K_y [m/day]

Inclination angle <x',x> β [deg]

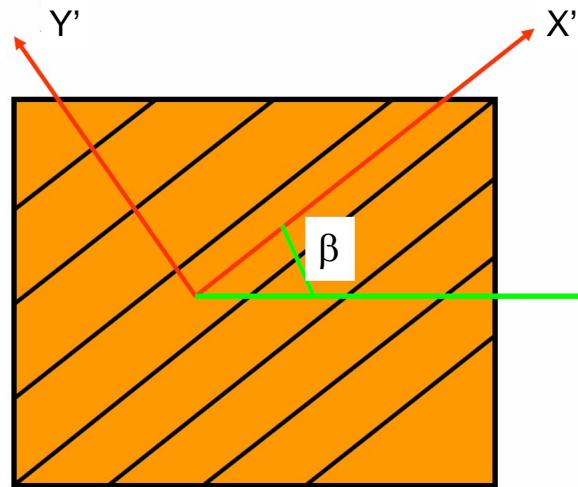
Saturation

Residual saturation ratio S_r

Saturation constant α [1/m]

Help OK Cancel

Data mode Advanced



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In 2D one may analyze isotropic or orthotropic flow problems. The main data set consists of fluid bulk modulus β^F that is set by default to a large value (quasi-incompressible case), seepage coefficients k'_x and k'_y along principal axes of permeability. These axes can be rotated with respect to the reference frame (X-Y) by angle β . Note that for $k'_x = k'_y$ isotropic flow conditions are fulfilled and orientation angle β becomes a meaningless parameter. The last two parameters α and S_r are responsible for modeling flow in the zone of partial saturation. By default $S_r = 0$ and $\alpha = 2.0$.

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Material data: Flow group for 3D problems

3D Flow

Fluid bulk modulus: [kN/m²]

Residual saturation S_r :

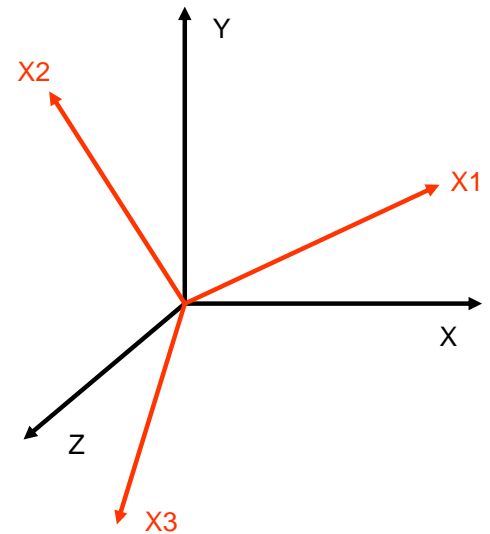
Seepage alpha α : [1/m]

Axis	k	Direction X	Direction Y	Direction Z
1	<input type="text" value="1"/> [m/day]	<input type="text" value="1"/>	<input type="text" value="0"/>	<input type="text" value="0"/>
2	<input type="text" value="1"/> [m/day]	<input type="text" value="0"/>	<input type="text" value="1"/>	<input type="text" value="0"/>
3	<input type="text" value="1"/> [m/day]	<input type="text" value="0"/>	<input type="text" value="0"/>	<input type="text" value="1"/>

A2[x] -> A2[y] -> A2[z] ->

Help Ok Cancel

Data mode: Advanced

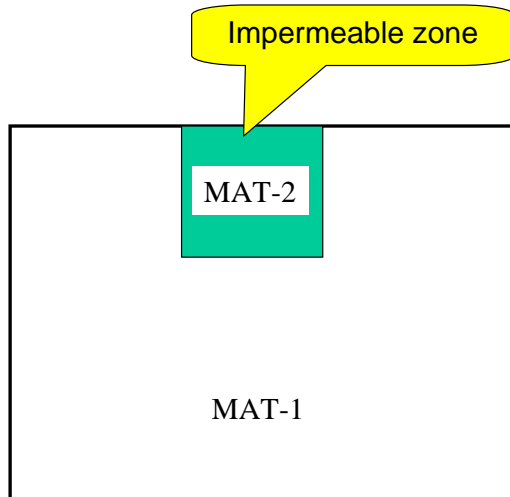


NB. Here we set the two vectors

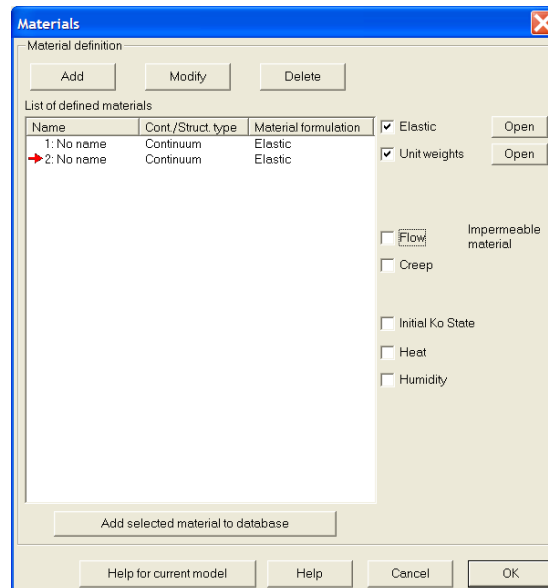
For 3D problems almost all flow parameters are set as for the 2D except the orientation of principal permeability axes. To set orientation of these axes only two of them (for the second one only two components are independent) can be defined while the 3-rd one is obtained automatically by a vector product.



Modeling impermeable materials: flow around impermeable zone



Setting impermeable material for continuum



Here we have to switch off the **Flow** group

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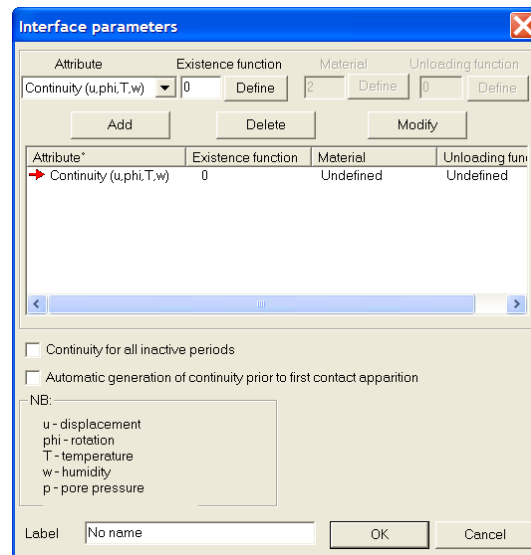
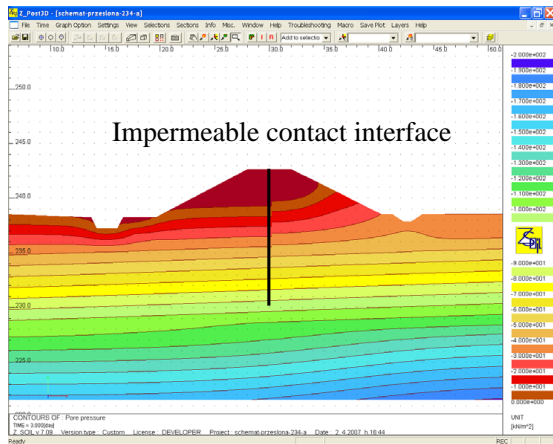
To model impermeable zones using steady state driver one cannot assume its very low permeability (zero value is forbidden) This is so because steady state gives a solution for an infinite time after the change of the flow regime. Only for transient flow it would be possible but switching off flow at the material level is definitely better.

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Modeling impermeable diaphragms

Adding interface elements with discontinuous pressure DOF

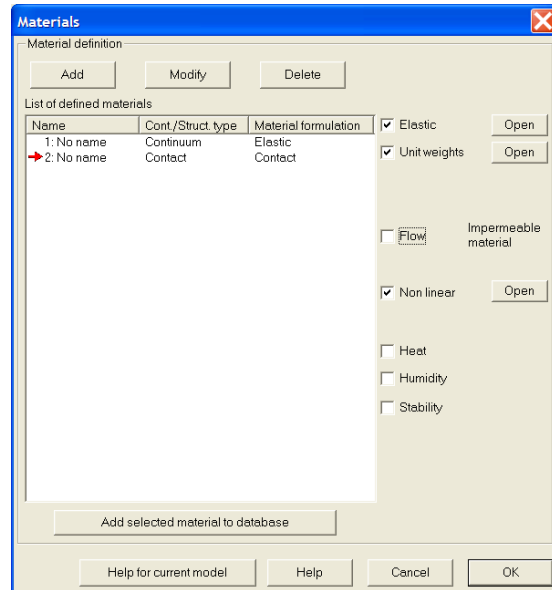
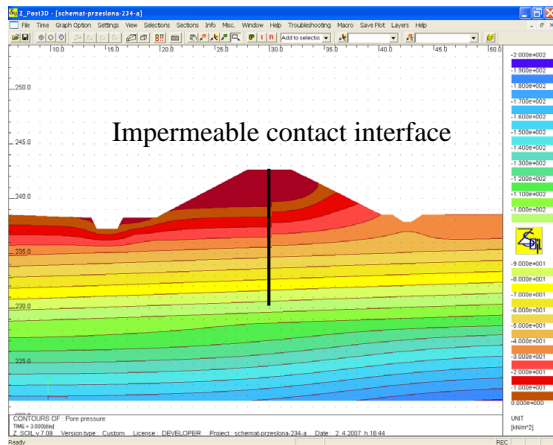


NB. This definition can be used when we assume that contact interface should generate *continuous* deformations while the interface zone remains impermeable

To analyze pressure distribution for problems with quasi-impermeable diaphragms one may assume that their permeability is practically equal to zero. Hence to discretize them one may use interface elements that may easily represent a discontinuous pressure field at the same point on both faces. In this example we can say that the interface will behave in such a manner that displacement field (if it is active) will be continuous while the pressure not (in this definition the interface will have an attribute *Continuity (u,phi,T,w)* (pressure is not mentioned in the list of compatible DOFs). With this definition user may change the interface permeability in time via existence functions.



Setting impermeable interface material



NB. This definition can be used when we assume that contact interface may generate *discontinuous* deformations and the interface zone remains impermeable

This definition is probably much easier than the previous one, but to modify the interface behavior user has to enforce hand restarts.



Modeling inflow of rain flux

Main problem: how to set the rain flux ?

- Simultaneous application of pressure BC and flux BC is not possible
- If we apply a flux to the boundary then as a reaction we obtain compressive pressure or suction
- If we apply pressure BC to the boundary then as reaction we obtain fluid flux (outflow)
- Note: rain flux cannot enter the domain if it is fully saturated; hence imposing flux on the boundary we may generate high compressive pressures that may lead to immediate loss of static equilibrium
- The easiest way to cut-off the flux is to apply it together with seepage surface but **flux \leq seepage coeff. !!!**
 - If soil is not fully saturated it will switch OFF the pressure BC, hence flux will be imposed
 - If soil is fully saturated it will switch ON the pressure BC, hence flux will be cancelled

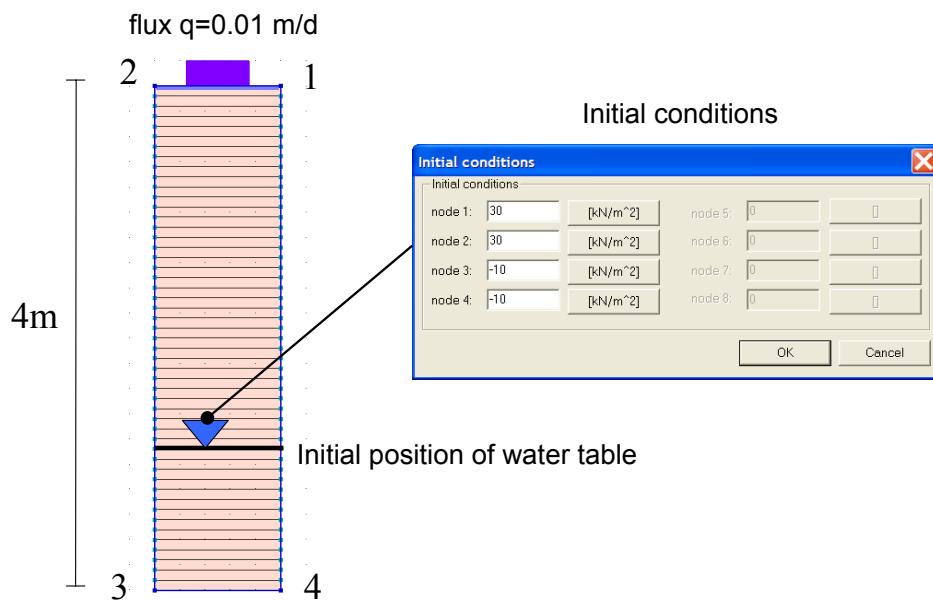
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Applying fluid flux will always produce an increase of pore pressure. In two-phase applications (consolidation) we have to be sure that soil may resist such a pressure build-up, otherwise solution will fail to converge.

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Modeling inflow of rain flux: Example



Data: $k_x = k_y = 0.01$ m/d, $\alpha = 2$, $S_r = 0$, $K^F = 200000$ kPa.

- This problem is run directly with Transient driver hence the initial pressures must be defined
- Note that except seepage surface no other boundary conditions are applied (it is allowed exclusively for transient case)

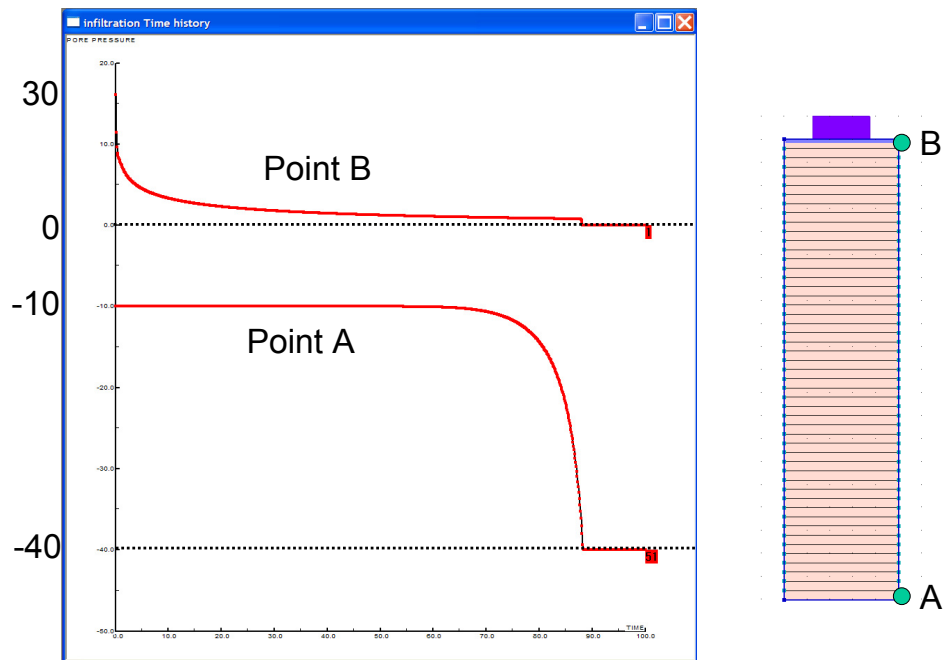
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Modeling infiltration (rain inflow for instance) process is the aim of this simple test. Here we do not need to set pressure BCs because we start directly with a transient driver. For given position of the phreatic surface the initial pressure distribution consists of compressive (-) pressures at the bottom and suction (+) at the top. The pressure distribution that satisfies the steady state equation is linear along vertical coordinate (between points 1 and 4, and 2 and 3).

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Modeling inflow of rain flux: Results



Pressure time history at selected points

Saturation profile

video

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In the above figure we can trace the time history of pressure distribution at points B (top) and A (bottom). We can see that the position of phreatic surface becomes almost unchanged until the full partially saturated zone becomes fully saturated.

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How to retrieve outflow time history ?

- 2D case **video**
- 3D case **video**

To compute the volume of the fluid flux one has to perform the following steps

- 1 go to postprocessor
- 2 make a cross section
- 3 switch to visualization of cross sectional quantities (*Graph analysis/Sectional quantities*)
- 4 under menu *Settings* select results for continuum → Fluid velocities
- 5 select normal (N) component to be visualized
- 6 under *Misc.* menu invoke *Sections/Integral* or *Sections/Time history of integrals*
- 7 save the output file in *.csv format and postprocess it using Excel



Seepage computations: troubleshooting

- Divergence of the iterative procedure
 - ① If it occurs for steady state driver try to carry out multi-steady state analysis (specifying variable in time boundary conditions and enforcing at least few computation steps)
 - ② If the above does not remedy the problem try to diminish α values (large α values may yield convergence problems (for $\alpha > 10.0$))
 - ③ Avoid too large differences of seepage coefficients among different materials ($1.0^{-6} \leq \frac{k_1}{k_2} \leq 1.0^6$); note that partial saturation diminishes permeability in unsaturated zone by factor 10^{-5}
- Pressure oscillations appear in the transient solution
 - ① Try to increase time steps but if this deteriorates the solution refine the mesh
- Compressive pressures appear at the free boundary which is to be permeable)
 - ① Add seepage elements